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Algorithm and Program to Design Machine-Tools Gears by Considering the Effect of Power and Torque Changes on the Load Rating

Gabriel Frumuşanu, Ştefan Mitu
University "Dunarea de Jos", Galati

ABSTRACT

The paper suggests an algorithm conceived to optimize the design of the gears from machine-tools gearboxes by considering the machine-tool working characteristics and the effect of power and torque changes on the load rating of machine-tool gears. The effect is taken into account in gear loading formulae by introducing the equivalent factor K_{eq}. Based on the special regularity of the load and speed changes of machine tools, the equivalent factor is analyzed and a theoretical calculation formula is suggested. Specific application of the algorithm and a dedicated soft are also shown.

1. Introduction

The calculation of the load capacity of machine-tool gears must be done, following ISO standards, [1], based on the formula

$$\sigma_F = \sigma_{FO} \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\alpha} \le \sigma_{FP}, \qquad (1)$$

where σ_F, σ_{FO} and σ_{FP} mean, respective, the calculated, the basic and the allowable stress and K_A, K_V, $K_{F\alpha}$, $K_{F\beta}$ - load factors, taking into account the effects of the loading condition prevailing in service.

Starting from here, we can observe that there is no factor which allows for the effect of load and speed changes on the calculated stress to be considered.

Generally speaking, when gears operate under variable load and running speed during their service life, failure of a gear is caused, usually, by the fatigue damage accumulation; machine-tool gears are making no exception to the rule. But machine-tools have most complex power and torque characteristics and their load and speed changes conform to a peculiar regularity. The characteristic features result in the difference of the rating of machine-tools gears from that of other gears.

Therefore, it becomes important to a designer to introduce a new load factor, taking into account the above effects in the formulae for the load rating of gears, having as purpose to make good design estimates for operation under conditions of spectrum loading and speed spectrum using the standards already existent.

This paper suggests the incorporation of a new load factor in gear rating, specific to machine-tool gearing – the equivalent factor, K_{eq}.

2. Machine-Tool Working Characteristic

In machine-tool cutting the used power is often changed. Figure 1 shows the power spectrum of an lathe engine [3], where the horizontal axis represents the ratio between the actual used power P_v and the maximum power P_u, and the vertical axis represents the power use time ratio α_v, which is defined as the ratio of the time spent at a given power to the total working time of the gear.

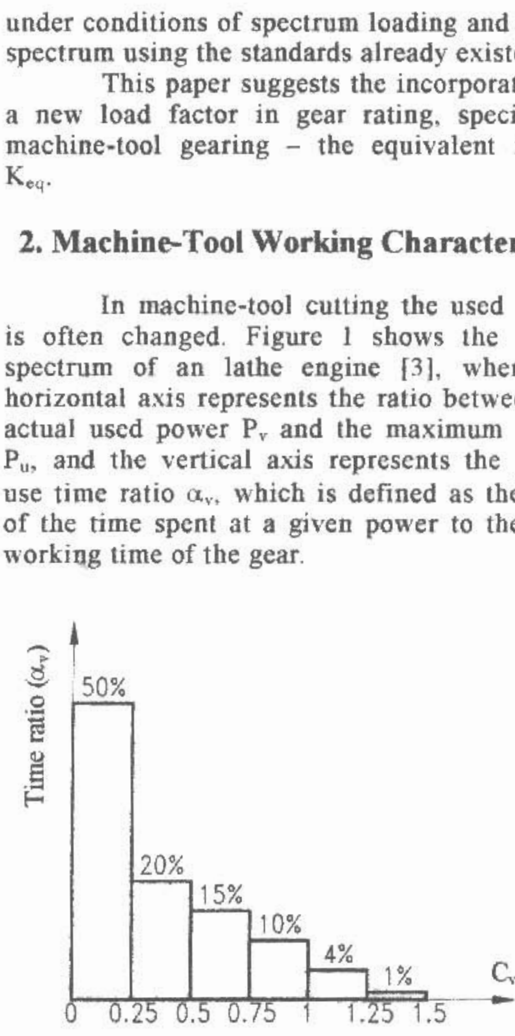

Fig. 1

The power use ratio C_v is defined as

$$C_v = P_v / P_u. \tag{2}$$

In addition to the different power use ratios, general machine-tools possess a large range of speed changes and more and different steps of speed. Figure 2 shows the time distributions of working speeds of the lathe [3], where the horizontal axis represents the spindle speed steps (e.g. 18 steps), and the vertical axis represents the corresponding time ratio, β_u, which is defined as the ratio of the time spent at a given step to the total working time.

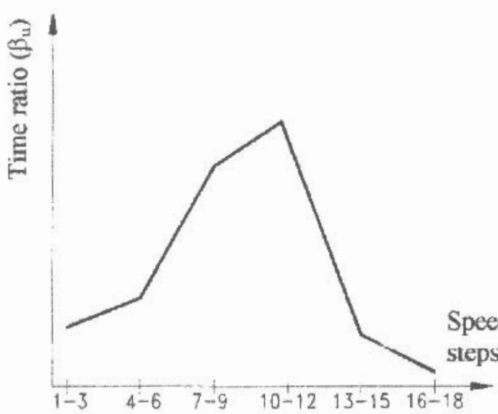

Fig. 2

3. The Equivalent Factor of Machine-Tools Gears

From the actual survey of the power use and the speed running-time distribution diagrams of the machine-tool, the load and speed change laws of the gears in the transmission chain are known.

Summing up the various working conditions of the gears, the actual load circulation diagram is obtained as shown in Fig.4, where the horizontal axis represents the number of cycles, N, the vertical axis represents the delivered torque, T and the broken line formed by T_1, T_2, ... T_k is the actual load, while N_1, N_2, ... N_k are the numbers of cycles corresponding to the loads.

Machine-tools gears have many speed steps in the working period, t. At one speed, different torques are transmitted and also each of the torques can be transmitted at different speeds.

If T_i represents an arbitrary torque from T_1 to T_k (Fig.3) and N_i an arbitrary number of cycles from N_1 to N_k, then N_i is the total number of cycles while the torque T_i is being transmitted.

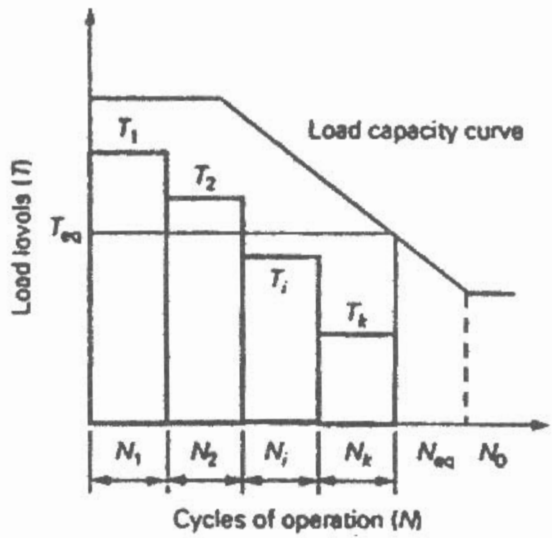

Fig.3

The fatigue result produced by the load circulation can be expressed by the equivalent torque T_{eq} and the equivalent number of cycles. The following equation is obtained:

$$T_{eq}^{\,p} \cdot N_{eq} = T_1^{\,p} \cdot N_1 + T_2^{\,p} \cdot N_2 + ... + T_k^{\,p} \cdot N_k = \sum_{i=1}^{k} T_i^{\,p} \cdot N_i \tag{3}$$

where p is the exponent of test material, and the equivalent number of cycles, N_{eq}, is the sum of the numbers of cycles at all speeds of a gear

$$N_{eq} = \sum_{i=1}^{k} N_i = \sum_{i=1}^{k} 60 \cdot n_i \cdot t_i\,, \tag{4}$$

where n_i is the gear speed (rot/min) and t_i is the working time (hours)corresponding to the gear speed n_i.

According to equations (4) and (3), the equivalent load T_{eq} becomes:

$$T_{eq} = \left[\sum_{i=1}^{k} \left(T_i^{\,p} \cdot N_i \right) \Big/ \sum_{i=1}^{k} N_i \right]^{1/p} = \left[\sum_{i=1}^{k} \left(T_i^{\,p} \cdot n_i \cdot t_i \right) \Big/ \sum_{i=1}^{k} \left(n_i \cdot t_i \right) \right]^{1/p}. \tag{5}$$

This expression shows the variable loading T_i and the speed n_i (or cycles N_i) to have a great effect on the equivalent load T_{eq}. The effect of the variable working conditions on the gear rating is here indicated by introducing a new load factor, termed the equivalent factor. Taking the highest load (torque) transmitted by the gears as the nominal load T, the equivalent factor can be defined as the ratio of the equivalent load to the nominal load and is denoted by K_{eq}. Then

$$K_{eq} = T_{eq} / T = \left[\sum_{i=1}^{k} (T_i / T)^p \cdot n_i \cdot t_i / \sum_{i=1}^{k} (n_i \cdot t_i) \right]^{1/p} =$$
$$= \left\{ \sum_{i=1}^{k} \left[(T_i / T)^p \cdot n_i \cdot t_i / (n_1 t) \right] \Big/ \left[\sum_{i=1}^{k} n_i \cdot t_i / (n_1 \cdot t) \right] \right\}^{1/p}, \tag{6}$$

where n_1 is the minimum speed of the gear (rot/min) and t is the total working time of the gear, in hours.

If a gear has z steps of speed and every one of them can transmit k kinds of different power and if the time ratio of using different power use of one speed is α_v ($v = 1, 2, \ldots k$), the time ratio of using the same power use at different speeds is β_u ($u = 1, 2, \ldots z$). On the supposition that they conform to the same distribution law under different power and speeds, the ratio between t_i and t under any working condition is

$$t_i / t = \alpha_v \cdot \beta_u . \tag{7}$$

Substituting equation (7) into equation (6), T_v and n_u instead of T_i and n_i, yields

$$K_{eq} = \left\{ \left[\sum_{v=1}^{k} \sum_{u=1}^{z} (T_v / T)^p (n_u / n_1) \cdot \alpha_v \cdot \beta_u \right] \Big/ \left[\sum_{v=1}^{k} \sum_{u=1}^{z} (n_u / n_1) \cdot \alpha_v \cdot \beta_u \right] \right\}^{1/p} \tag{8}$$

This equation gives the general form of the equivalent factor, K_{eq}, for machine tool gears under changing power and rotational speed.

In design , the factor K_{eq} can be used to modify the calculated stress in the equation for load rating of gears. After calculating K_{eq}, the equation (1) becomes

$$\sigma_F = \sigma_{FO} \cdot K_{eq} \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\alpha} \le \sigma_{FP} \tag{9}$$

the new factor K_{eq} being added to the existing load factors.

As it is well known, the basic stress σ_{FO} is calculated by the nominal load T. Multiplying σ_{FO} by K_{eq} means that the basic stress is determined by the equivalent load, T_{eq}, thereby correcting the calculated stress σ_F.

The equivalent number of cycles, N_{eq}, directly affects an allowable stress in the load rating equations. For example, neglecting other factors, the relation of the allowable stress σ_{FO} in (1) to N_{eq} may be expressed as (refer to Fig.3)

$$\sigma_{FP} = \sigma_{lim} \left(N_0 / N_{eq} \right)^{1/p} . \tag{10}$$

The equation (10) shows that N_{eq} increases the allowable stress from the fatigue limit, σ_{lim}, corresponding to the basic number of cycles, N_0, to the value of stresses corresponding to itself on fatigue curves.

4. Algorithm and Calculus Program to Find K_{eq}

Because formula to calculate the equivalent factor are complicated, we considered that an easier way to do it is welcome.

Based on the equations upper exposed an algorithm and a program to calculate the equivalent factor were especially designed.

The following input data are necessary to start the calculus:

- the number of power use steps, k;
- the coefficients giving the power use time ratio, α_v, $v = 1, 2, \ldots k$;
- the ratio of the principal driving chain speeds, φ;
- the exponent of the material test, p;
- the number of principal driving chain speeds groups, z;
- the coefficients giving the speeds-groups use time ratio, β_u, $u = 1, 2, \ldots z$;
- the working regime of the designed gear (constant power range, constant torque range or mixed).

Based on these values, the program calculates by iterations the sums from (9) and gives the value for K_{eq}.

Finally, the question if the user wants to run once again the calculus is asked.

5. Numerical Results

By using the calculus program upper presented, many cases of gearboxes gears loading were considered, in order to study the influence of various factors on the equivalent factor; for example, we can mention the following results:

- if $k = 6$, $\alpha_1 = 0.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.15$, $\alpha_1 = 0.1$, $\alpha_5 = 0.04$, $\alpha_6 = 0.01$, $\varphi = 1.32$, $p = 6.6$, $z = 6$, $\beta_1 = 0.1$, $\beta_2 = 0.15$, $\beta_3 = 0.3$, $\beta_4 = 0.35$, $\beta_5 = 0.08$, $\beta_6 = 0.02$, all the z steps of speeds in the constant power range, then $\mathbf{K_{eq} = 0.778}$;
- if $k = 6$, $\alpha_1 = 0.55$, $\alpha_2 = 0.25$, $\alpha_3 = 0.1$, $\alpha_1 = 0.06$, $\alpha_5 = 0.03$, $\alpha_6 = 0.01$, $\varphi = 1.26$, $p = 6.6$, $z = 6$, $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\beta_3 = 0.3$, $\beta_4 = 0.2$, $\beta_5 = 0.15$, $\beta_6 = 0.05$, all the z steps of

speeds in the constant power range, then $K_{eq} = 0.811$;

- if k = 5, $\alpha_1 = 0.6$, $\alpha_2 = 0.25$, $\alpha_3 = 0.1$, $\alpha_4 = 0.04$, $\alpha_5 = 0.01$, $\varphi = 1.26$, p = 6.6, all the z steps of speeds in the constant torque range, then $K_{eq} = 0.943$;

6. Conclusions

The results obtained, in all the analyzed situations considered, are showing that the optimization of gear design, based on effective power and torque characteristics of machine-tools, leads to diminutions (sometimes substantial) of the gear teeth dimensions.

Thus, the dimensions of the gear boxes can be reduced without affecting their the loading capacity.

References

[1] – **ISO/DIS 6336**, *Calculation of Load Capacity of Spur and Helical Gears*, ISO/TC60/WG6 (1987).

[2] – **D. Reshetov**, *Elements and Mechanisms of Metal Cutting Machine-Tools*, Nauk, Moskow, 1972.

[3] – **J. Yazhou, Z. Di**, *A Note on the Rating of Gears Based on Power and Torque Characteristics of Machine Tools*, J. Machine Tools Manufacturing, Vol.32, No.5, p.659-669, 1992.

[4] – **Şt. Mitu** ş.a. – *Maşini-unelte. Principii de proiectare*, Ed. Tulias, Constanţa. 1995.

ALGORITM ŞI PROGRAM PENTRU DIMENSIONAREA ANGRENAJELOR CUTIILOR DE VITEZE ÎN FUNCŢIE DE ÎNCĂRCAREA REALĂ A ACESTORA

Rezumat

În lucrare este vorba despre efectul caracteristicilor temporale de utilizare a maşinilor unelte, cumulat cu cel al schimbărilor puterii şi momentului de torsiune necesare diferitelor prelucrări, asupra dimensionării raţionale a angrenajelor cutiilor de viteze. Efectul poate fi luat în calcul prin intermediul unui coeficient de corecţie aplicat în formula de dimensionare a angrenajelor pe baza solicitării la uzura de contact. Pentru uşurarea calculului coeficientului de corecţie au fost concepute un algoritm şi un program de calcul; exemple de aplicare a acestora sunt, de asemenea, prezentate.

ALGORITHME ET PROGRAMME POUR DIMENSIONER LES ENGRENAGES DES BOITS DE VITESSES POUR LES MACHINES-OUTILS, EN TENANT COMPTE DU CHARGEMENT REEL

Résumé

Dans ce papier il s'agit de l'effet des caractéristiques temporels concernant l'usage des machines-outils, cumulé avec celui des changements de la pouvoir et du moment de torsion nécessaires pour les différentes usinages, sur le dimensionnement rationnel des engrenages des boites de vitesses. L'effet peut-être compris par l'intermède d'un coefficient de correction, appliqué dans la formule utilisée pour dimensionner les engrenages en tenant compte de la sollicitation a la pression du contact. Pour rendre plus facile le calcul du coefficient de correction, un algorithme et un programme de calcul ont été imaginés ; des exemples pour leur application sont, aussi, présentés.